OCR

Monday 16 June 2014 – Morning

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Evaluate $\int_0^{\frac{1}{6}\pi} (1 - \sin 3x)\,\mathrm{d}x$, giving your answer in exact form. **[3]**

2 Find the exact gradient of the curve $y = \ln(1 - \cos 2x)$ at the point with x-coordinate $\frac{1}{6}\pi$. **[5]**

3 Solve the equation $|3 - 2x| = 4|x|$. **[4]**

4 Fig. 4 shows the curve $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = a + \cos bx,\ 0 \leqslant x \leqslant 2\pi,$$

and a and b are positive constants. The curve has stationary points at $(0, 3)$ and $(2\pi, 1)$.

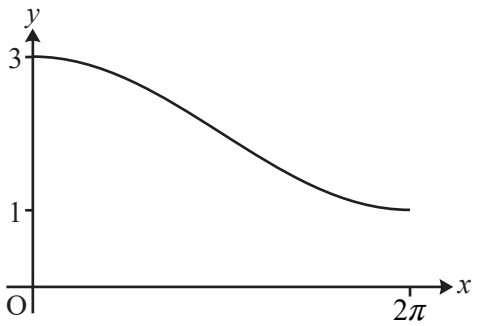

Fig. 4

(i) Find a and b. **[2]**

(ii) Find $\mathrm{f}^{-1}(x)$, and state its domain and range. **[5]**

5 A spherical balloon of radius r cm has volume $V\,\mathrm{cm}^3$, where $V = \frac{4}{3}\pi r^3$. The balloon is inflated at a constant rate of $10\,\mathrm{cm}^3\,\mathrm{s}^{-1}$. Find the rate of increase of r when $r = 8$. **[5]**

6 The value £V of a car t years after it is new is modelled by the equation $V = A\mathrm{e}^{-kt}$, where A and k are positive constants which depend on the make and model of the car.

(i) Brian buys a new sports car. Its value is modelled by the equation

$$V = 20000\,\mathrm{e}^{-0.2t}.$$

Calculate how much value, to the nearest £100, this car has lost after 1 year. **[2]**

(ii) At the same time as Brian buys his car, Kate buys a new hatchback for £15000. Her car loses £2000 of its value in the first year. Show that, for Kate's car, $k = 0.143$ correct to 3 significant figures. **[3]**

(iii) Find how long it is before Brian's and Kate's cars have the same value. **[3]**

7 Either prove or disprove each of the following statements.

(i) 'If m and n are consecutive odd numbers, then at least one of m and n is a prime number.' **[2]**

(ii) 'If m and n are consecutive even numbers, then mn is divisible by 8.' **[2]**

Section B (36 marks)

8 Fig. 8 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{x}{\sqrt{2+x^2}}$.

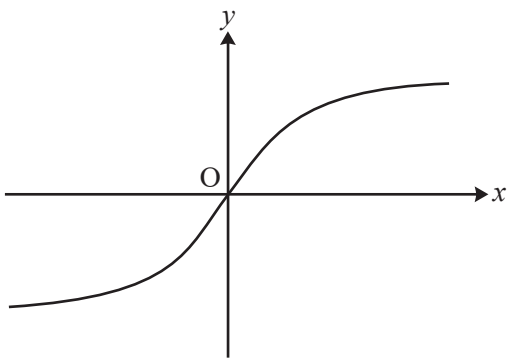

Fig. 8

(i) Show algebraically that $\mathrm{f}(x)$ is an odd function. Interpret this result geometrically. **[3]**

(ii) Show that $\mathrm{f}'(x) = \dfrac{2}{(2+x^2)^{\frac{3}{2}}}$. Hence find the exact gradient of the curve at the origin. **[5]**

(iii) Find the exact area of the region bounded by the curve, the x-axis and the line $x = 1$. **[4]**

(iv) (*A*) Show that if $y = \dfrac{x}{\sqrt{2+x^2}}$, then $\dfrac{1}{y^2} = \dfrac{2}{x^2} + 1$. **[2]**

(*B*) Differentiate $\dfrac{1}{y^2} = \dfrac{2}{x^2} + 1$ implicitly to show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2y^3}{x^3}$. Explain why this expression cannot be used to find the gradient of the curve at the origin. **[4]**

[Question 9 is printed overleaf.]

9 Fig. 9 shows the curve $y = xe^{-2x}$ together with the straight line $y = mx$, where m is a constant, with $0 < m < 1$. The curve and the line meet at O and P. The dashed line is the tangent at P.

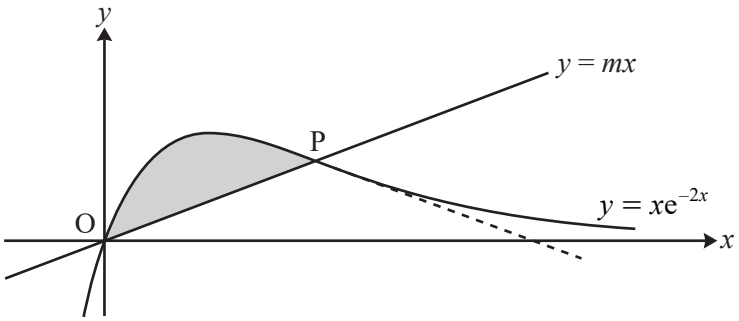

Fig. 9

(i) Show that the x-coordinate of P is $-\frac{1}{2}\ln m$. **[3]**

(ii) Find, in terms of m, the gradient of the tangent to the curve at P. **[4]**

You are given that OP and this tangent are equally inclined to the x-axis.

(iii) Show that $m = e^{-2}$, and find the exact coordinates of P. **[4]**

(iv) Find the exact area of the shaded region between the line OP and the curve. **[7]**

END OF QUESTION PAPER